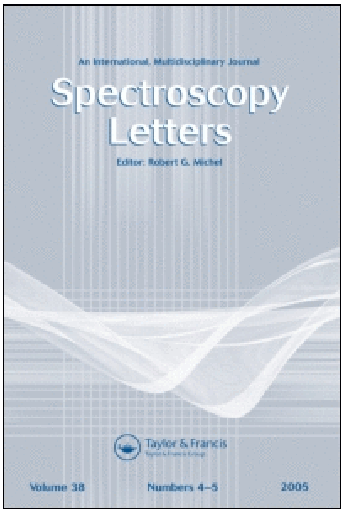

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Non-Standard Normalization of the Second-Order Factorial Moment as a Method of Reducing the Error Involved in Characterizing Spectral Profiles

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NON-STANDARD NORMALIZATION OF THE SECOND-ORDER FACTORIAL MOMENT AS A METHOD OF REDUCING THE ERROR INVOLVED IN CHARACTERIZING SPECTRAL PROFILES.

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ABSTRACT.

The use of non-standard normalized second-order factorial moment may reduce the error involved in analyzing a light signal.

In this work we derive the theoretical expression of the error obtained when this kind of functions is used.Experimental results for a periodic square wave-shape profile are given.

1.INTRODUCTION

The measurement of statistical function is a powerfull tool for analyzing a weak light beam.The error involved in determining the parameters characterizing a light beam depends widely on the statistical function which is measured. The most frequently used is the autocorrelation function $g^{(2)}(\tau)$ which is evaluated for a set of delay time (τ_i). This function is well-related[1] with the second-order factorial moment $n^{(2)}(T)$ which may be evaluated for a set of values of the cuonting time (T_i). We obtain similar errors by fitting the experimental values of $g^{(2)}(\tau_i)$ or $n^{(2)}(T_i)$ to the corresponding theoretical ones[2]. However $n^{(2)}(T)$ present the additional adventage of its simple measurement methods.

The measurement of $n^{(2)}(T)$ has been applied succefully to analyze periodic[3] and no-deterministic[4] light.Here appear the posibility of defining two new functions (n_1,n_3) more exacts,based on $n^{(2)}(T)$ because of its simple measurement procedure.

In this paper we develop the theoretical expression of two new functions in order to compare they with the already known $n^{(2)}(T)$. This expressions are useful to predict the experimental error involved in determining the characterizing parameters of a light beam and to determine the best experimental conditions.

2.METHOD

The normalized second-order factorial moment is defined as:

$$n^{(2)}(T)=\frac{\langle n\ (n-1)\rangle}{\langle n\rangle^2}=n_2 \qquad (1)$$

Where n is the number of photocounts per connting time T.

We define the functions n_1 and n_3 as follows:

$$n_1\ (T)\ =\frac{\langle\ n\ .\ (\ n-1\ )\rangle}{\langle\ n\ \rangle} \qquad (2)$$

$$n_3\ (T)\ =\frac{\langle\ n\ .\ (\ n-1)\rangle}{\langle\ n\rangle^{\ 3}} \qquad (3)$$

Such that $n^{(2)}\ (T) = (\ n_1\ (T)\ .n_3\ (T)\ )^{\frac{1}{2}}$ .If we derive the Var $n^{(2)}\ (T)$ we obtain:

$$\text{Var}\ n^{(2)}\ (T) \simeq \tfrac{1}{2}\ (\ \frac{\text{Var}\ n_1}{\bar{n}^{\ 2}}\ +\ \bar{n}^2\ \text{Var}\ n_3\ ) \qquad (4)$$

In Eq (4) we can see that when $\bar{n} \gg 1$ the value of Var n_3 must be very small to keep the value of Var n_2 from being too large.The same thing occurs with Var n_1 when $\bar{n} \ll 1$.

The possibility of obtaining small values of Var n_1 and Var n_3 encouraged us to derive the expression of these variances in order to evaluate the error that can be obtained by measuring the functions $n_1(T)$ and $n_3(T)$.

The expresions of the variances were derived by the same method that was used for Var$n^{(2)}(t)$ in Ref.(5) and for Var$g^{(2)}(\tau)$ in Ref.(6):

$$\mathrm{Var}\ n_1(T) = \left(\frac{1}{\bar{n}}\right)^2 \mathrm{Var}\,(N^{(2)}) + \left(\frac{N^{(2)}}{\bar{n}^2}\right)^2 \mathrm{Var}\,(\bar{n}) + 2\left(\frac{-N^{(2)}}{\bar{n}^3}\right)\left(\langle N^{(2)} n\rangle - \bar{n}\, N^{(2)}\right) \qquad (5)$$

$$\mathrm{Var}\ n_3(T) = \left(\frac{1}{n}\right)^2 \mathrm{Var}\,(N^{(2)}) + \left(\frac{-3N^{(2)}}{\bar{n}^4}\right) \mathrm{Var}\,(\bar{n}) + 2\left(\frac{-3N^{(2)}}{\bar{n}^7}\right)\cdot\left(\langle N^{(2)}\bar{n}\rangle - \bar{n}\langle N^{(2)}\rangle\right) \qquad (6)$$

Where

$$\mathrm{Var}\ N^{(r)} = \frac{\bar{n}^{2r}}{N}\left(\sum_{s=0}^{r} s!\binom{r}{s}^2 \frac{n^{(2r-s)}}{\bar{n}^{s}} - n^{(r)2}\right) \qquad (7)$$

and

$$\langle N^{(2)} n\rangle = \frac{1}{N}\left(N^{(3)} + 2N^{(2)} + (N-1)\,\bar{n}\,N^{(2)}\right) \qquad (8)$$

where N is the number of samples and $N^{(r)}$ the r-order factorial moment.

Using Eq (5),(6),(7),and (8) we obtain

$$\mathrm{Var}\ n_1 = \frac{\bar{n}^2}{N}\left(n^{(4)} + n^{(3)}\left(\frac{4}{\bar{n}} 2n^{(2)}\right) + 2\frac{n^{(2)}}{\bar{n}^2} + n^{(2)2}\left(n^{(2)} - \frac{3}{\bar{n}}\right)\right) \qquad (9)$$

$$\mathrm{Var}\ n_3 = \frac{1}{\bar{n}^2 N}\left(n^{(4)} + n^{(3)}\left(\frac{4}{\bar{n}} - 6n^{(2)}\right) + 2\frac{n^{(2)}}{\bar{n}^2} + n^{(2)2}\left(9n^{(2)} - \frac{3}{\bar{n}} - 4\right)\right) \qquad (10)$$

Equations (9) and (10) together with the expression corresponding to n_2:

$$\mathrm{Var}\ n_2 = \frac{1}{N}\left(n^{(4)} + n^{(3)}\left(\frac{4}{\bar{n}} - 4\, n^{(2)} \right) + 2n^{(2)}/\,\bar{n}^2 + n^{(2)^2}\left(4n^{(2)} - 4\,/\,\bar{n} - 1\right)\right) \tag{11}$$

Will allow us to evaluate the error with which we may obtain a parameter characterizing a light beam α , when n_1 ,n_2 or n_3 is measured.

It we want to estimate error we must use the expression:

$$\mathrm{Var}\ \alpha = \left(\sum_{i=1}^{L} \left(\frac{\partial n_j(T_i)}{\partial \alpha}\right)^2 \right)^{-2} \left(\sum_{i=1}^{L} \left(\frac{\partial n_j(T_i)}{\partial \alpha}\right)^2 \mathrm{Var}\ n_j(T_i) \right),\ j=1,2,3 \tag{12}$$

Where the source has a coherence time much smaller than the counting times T_i.

We can see that the factor $\bar{n}^2$ appears in Eq(9) whereas in Eq(10) the factor $1/\ \bar{n}^2$ appears.These factor will produce small values of Var n_1 ($\bar{n} \ll 1$) or small values of Var n_3 ($\bar{n} \gg 1$) depending on the mean intensity of the light beam.

3.EXPERIMENTAL CHECKING.

The experimental checking is performed whit the aim of verifying the theoretical approximations made to obtain the error expressions.

There is a situation of special interest in which the factor $\bar{n}$ appear in the derivative $\partial n_3/\partial\alpha$ and the factor $1/\ \bar{n}$ appears in $\partial n_1/\partial\alpha$ so that the theoretical advantage previously mentioned may be cancelled out.This allows us to compare the three statiscal functions under the worst possible conditions for n_1 and n_3.

To do this we compare the theoretical and experimental results when using a light source (for exemple a LED) fed by a variable voltage with a square - waveshape.Since the coherence time of the light emitted by a LED is $\tau_c \sim 10^{-13}$ sec and counting time T was $>10^{-3}$, the condition that $T \gg \tau_c$ is fulfilled .

The normaliced r-order factorial moment n_1 and n_2 are defined as

$$n^{(r)}(T) = \frac{\langle W^r (T) \rangle}{\langle W \rangle^r}$$

$$n_1 (T) = \frac{\langle w^2 (T) \rangle}{\langle w \rangle}$$

$$n_3 \quad T = \frac{\langle w^2 (T) \rangle}{\langle w \rangle^3} \qquad (13)$$

Where

$$W (ti,T) = \int_{ti}^{ti+T} I (t') dt'$$

In this case W(ti,T) takes the following form:

$$W \begin{cases} AT & 0 < t \leq p/2 - T \\ A(P/2-t) & P/2 - T < t \leq P/2 \\ 0 & P/2 < t \leq P-T \\ A(t+T-P) & P-T < t \leq P \end{cases} : T < P/2 \; ; \quad \begin{cases} A(P/2) & 0 < t \leq P-T \\ A(T-P/2) & P-T < t \leq P/2 \\ A(t+T-P) & P/2 < t \leq \frac{3P}{2} - T \\ A\ P/2 & \frac{3P}{2} - T < t \leq P \end{cases} : T > P/2$$

(14)

By using Eq 13,14,9,10,11,12, we can deduce the theoretical error involved in the process of experimental measurement.

The experimental arrangement is shown in Fig 1.Light source S consists of a controlled LED fed by functions generator F G producing a square- wave whose period is measured with the counter C .A pulse generator P G determi nes the counting time.The computer takes the counts from the counter,which is fed by a photomultiplier (Ph) and an amplifier- discriminator (A/D) and processes the data to obtain an experimental error value.

The experimental conditions were the following:

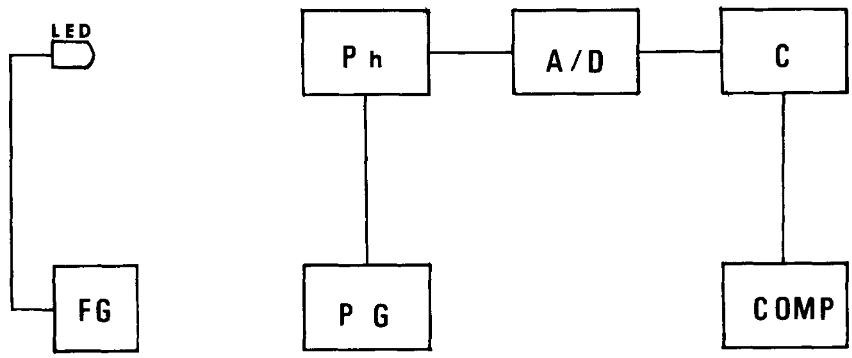

Fig. 1.- Experimental set-up

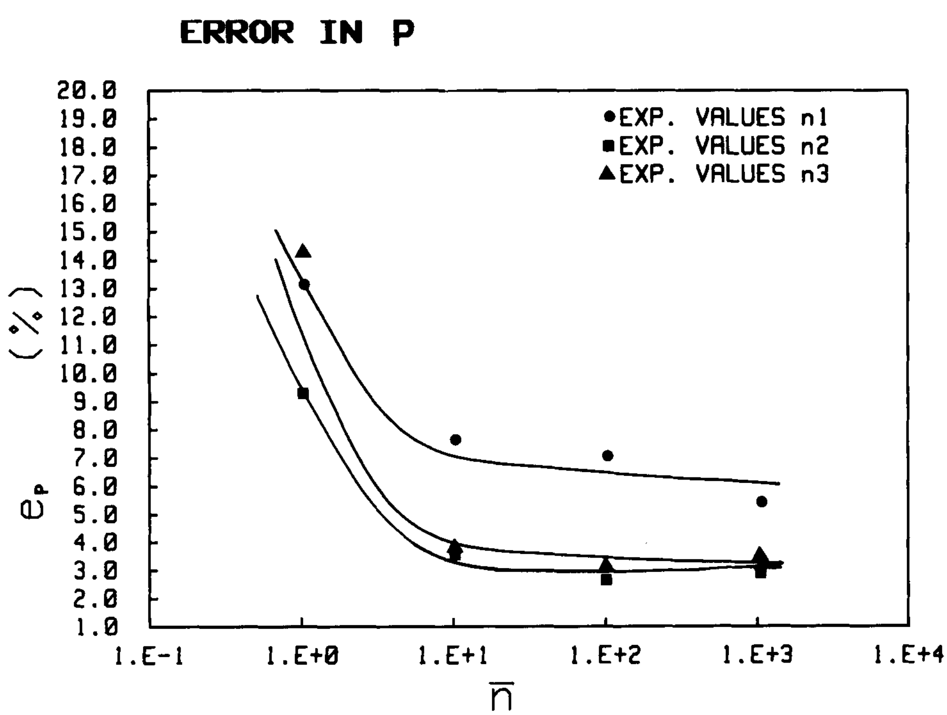

Fig. 2.- Theoretical and experimental values of e_{p1},e_{p2} and e_{p3} for different values of $\bar{n}$.

$P = 10^{-13}$ sec and the counting time $T \leqslant 0.5 \times 10^{-3}$ sec.The number of samples of W(T) taken was N = 1000 and the number of values of n_i(i= 1,2,3) used to obtain the error was equal to ten.

Figure 2 shows the theoretical and experimental relative error,in percent involved in determining the period (ep) by measuring n_i(i= 1,2,3) for different values of the number of photocounts per counting time $\bar{n}$.

If we compare the three statistical functions we can see that the use of n_3 results in errors a little larger than n_1 and n_2.We also observe that the errors decrease as value of $\bar{n}$ increases.

4.CONCLUSION

We can observe that there is a good agreement between the theoretical and experimental results ,confirming the validity of the develop made to obtain the theoretical expressions.

Hence,we can expect that for small values of the light intensity the use of $n_1(T)$ may show,depending on the parameter to be fitted, advantage over the use of $n^{(2)}(T)$.For great values of the light intensity is $n_3(T)$ which present advantage.

Figure 2 shows that under the most adverse conditions for $n_1(T)$ and $n_3(T)$ the three statistical functions present similar results.

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